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Magnetic moments and magnetic dipole transitions in deformed nuclei

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Abstract. The magnetic moments of the lowest rotational states of even–even nuclei and the probabilities of magnetic dipole transitions between these states are calculated on the basis of the model suggested by Krutov in 1968, the non-axiality of the nuclear equilibrium shape and the difference between the mass and charge distributions being taken into account. The results of calculations are compared with the available experimental results, the agreement being reasonably satisfactory.

1. Introduction

The magnetic moments of the rotational states of even–even nuclei and the magnetic dipole transitions between these states are described at present in a number of rotational models (Bohr and Mottelson 1953, Davydov and Filippov 1959, Nilsson and Prior 1961, Greiner 1966), all of them being based either on the suggested analogy of the nucleus with the ideal liquid drop (Bohr 1952, Bohr and Mottelson 1953) or on the assumption of Inglis's 'cranking model' (Inglis 1954, 1956). In the simple rotational model of Bohr and Mottelson the gyromagnetic factors of the rotational states are equal to Z/A. Nilsson and Prior (1961) obtained results differing from Z/A, the magnetic moments being calculated on the basis of the 'cranking model'. In their paper the difference of the neutron and proton pairing constants was taken into account. The difference of the neutron and proton distributions was derived by Greiner (1965, 1966) from the discrepancy between these constants using the 'quasi-spin' model (Kerman 1961); the magnetic moments of the rotational states were calculated on this basis in the framework of the Bohr–Mottelson hydrodynamic formalism, somewhat modified by Faessler *et al.* (1964). The magnetic dipole transitions between the rotational states with $I^{\pi}K = 2^{+}2$ and $I^{\pi}K = 2^{+}0$ were considered by Greiner (1965, 1966) in the same way. It is worth noting that the admixing of M1 transitions with E2 transitions is impossible in the simple rotational model. To derive the M1 admixture Davydov and Filippov (their model too is based on the analogy with the liquid drop) had to consider terms of the second order of smallness in the magnetic dipole moment operator†.

In the present paper the magnetic moments of the rotational states are considered on the basis of the rotational model suggested by Krutov (1968 a, b, to be referred to as I and II respectively). In our approach the non-axiality of the nuclear equilibrium shape and the difference in mass and charge distributions in the nucleus is taken into account. The difference between the parameters of the total deformation of mass and charge was calculated from the positions of the first levels in even–even nuclei with $I^{\pi}K = 1^{-}0$, using the 'charge oscillation model' (Krutov 1968 c, to be referred to as III). The mass and charge non-axiality parameters were derived, respectively, from the energies of the first two rotational levels with $I^{\pi} = 2^{+}$ and from the ratios of the reduced probabilities of E2 transitions into these levels from the ground state (Krutov and Zackrevsky 1969, to be referred to as IV). Using these values the authors calculated the gyromagnetic factors of the rotational states with $I^{\pi}K = 2^{+}0$ and $I^{\pi}K = 2^{+}2$ and the probabilities of M1 transitions between these states for a number of even–even nuclei. (The M1 admixture appeared in our approach owing to the difference between the charge and mass non-axiality parameters.) The results of our calculations are compared with the available experimental data and with the calculations by Greiner (1965, 1966).

† This approach is doubtful as it is based on the fact that the angular momentum and magnetic dipole moment are calculated with different accuracy. As is shown by Lipas (1964), the M1 admixture disappears when both the angular momentum and the magnetic dipole moment are calculated with an accuracy up to terms of the second order in the deformation parameter.

2. Magnetic moments of the rotational states of even–even nuclei

The magnetic moment of a system of charge, distributed with density $\rho^e(\boldsymbol{r}')$ and rotating with angular velocity $\boldsymbol{\omega}$, is given by

$$\boldsymbol{\mu} = \frac{e}{2mc}\int \{\boldsymbol{r}' \times (\rho^e \boldsymbol{\omega} \times \boldsymbol{r}')\}\, \mathrm{d}\boldsymbol{r}' \tag{1}$$

$\boldsymbol{r}'$ being the coordinate in the fixed-body system. This expression for $\boldsymbol{\mu}$ is similar to that for the angular momentum $\boldsymbol{I}$ with the only difference that $\rho(\boldsymbol{r}')$ is replaced by $(e/2mc)\rho^e(\boldsymbol{r}')$. In accordance with the model suggested in I only the 'moving density' (instead of $\rho(\boldsymbol{r}')$) should be taken into account in the expressions for the moments of inertia ($J_\nu = \hbar \boldsymbol{I}_\nu/\boldsymbol{\omega}_\nu$). Its components are equal to

$$\tilde{\rho}_\nu(\boldsymbol{r}') = \rho(\boldsymbol{r}') - \{\rho_{\mathrm{min}}(\boldsymbol{r}')\}_\nu \tag{2}$$

where $\{\rho_{\mathrm{min}}(\boldsymbol{r}')\}_\nu$ is the minimum density under the rotation around the ν axis.

Evidently a similar replacement should also be carried out in $\rho^e(\boldsymbol{r}')$ when $\boldsymbol{\mu}$ is considered. Then we have

$$\boldsymbol{\mu}_\nu = \mu_{\mathrm{B}} \frac{J_\nu^{\,e}}{J_\nu} \boldsymbol{I}_\nu = \mu_{\mathrm{B}} g_\nu \boldsymbol{I}_\nu \tag{3}$$

where

$$J_\nu^{\,e} = \int \tilde{\rho}_\nu^{\,e}\{r^2 - (x_\nu')^2\}\, \mathrm{d}\boldsymbol{r}' \tag{4a}$$

$$\tilde{\rho}_\nu^{\,e}(\boldsymbol{r}') = \rho^e(\boldsymbol{r}') - \{(\rho^e_{\mathrm{min}}(\boldsymbol{r}')\}_\nu \tag{4b}$$

$$J_\nu = \int \tilde{\rho}_\nu\{r^2 - (x_\nu')^2\}\, \mathrm{d}\boldsymbol{r}' \tag{4c}$$

and $\mu_{\mathrm{B}} = e\hbar/2mc$, the nuclear magneton.

We shall note that the value of $J_\nu^{\,e}$ so defined can be only conditionally called the proton 'moment of inertia', since the effective moment of inertia J_ν, as is seen from the expressions for $\tilde{\rho}_\nu$ and $\tilde{\rho}_\nu^{\,e}$, is not the sum of the corresponding values for the protons and neutrons of the nucleus.

For the practical calculations we shall assume the mass distribution to be represented by an ellipsoid with a sharp boundary, and we shall consider the form of the charge distribution to be the same. (The only values which are different are the deformation parameters and the equivalent radii of distribution.) Using the formulae (4*a*) and 4(*b*) for the moments of inertia from IV, we obtain the following expressions for the g_ν factors:

$$g_{1,2} = \frac{Z}{A}\left(\frac{R_{\mathrm{e}}}{R}\right)^2 \left\{1 - \frac{\beta-\beta_{\mathrm{e}}}{\beta} + \frac{31}{112}\left(\frac{5}{\pi}\right)^{1/2} \frac{\beta-\beta_{\mathrm{e}}}{\beta}\beta_{\mathrm{e}} \pm \frac{\beta_{\mathrm{e}}}{\beta\sqrt{3}}(\gamma_{\mathrm{e}}-\gamma) + \ldots\right\}$$
$$\simeq \frac{Z}{A}\left(\frac{R_{\mathrm{e}}}{R}\right)^2 \frac{\beta_{\mathrm{e}}}{\beta} \tag{5a}$$

$$g_3 = \frac{Z}{A}\left(\frac{R_{\mathrm{e}}}{R}\right)^2 \frac{\beta_{\mathrm{e}}}{\beta}\frac{\gamma_{\mathrm{e}}}{\gamma}\left\{1 + \frac{4}{7}\left(\frac{5}{\pi}\right)^{1/2}(\beta-\beta_{\mathrm{e}}) + \ldots\right\} \tag{5b}$$

where β and β_{e} are the total deformation parameters, γ and γ_{e} are the parameters of non-axiality, R and R_{e} are the equivalent distribution radii (respectively for mass and charge).

The gyromagnetic factor g_{R} for the $|IK\rangle$ rotational state is found to be the mean value of the operator

$$\hat{\mu} = \frac{\boldsymbol{\mu} \cdot \boldsymbol{I}}{I+1} = \frac{\sum_\nu \boldsymbol{\mu}_\nu \boldsymbol{I}_\nu}{I+1}. \tag{6}$$

Neglecting small admixtures of states with $K' \neq K$ (cf. II and IV), i.e. using the symmetric-top wave functions $\Phi_{MK}{}^I$ as the rotational ones, we obtain

$$g_{\mathrm{R}}(IK) \simeq \frac{1}{I}\langle \Phi_{MK}{}^I | \hat{\mu} | \Phi_{MK}{}^I \rangle$$

$$= \frac{g_1+g_2}{2}\left\{1 - \frac{K^2}{I(I+1)}\right\} + g_3 \frac{K^2}{I(I+1)}. \tag{7}$$

The difference between the parameters of mass and charge deformations necessary for the calculation of g_ν may be determined in principle from the values of the moments of inertia and the quadrupole moments of the nuclei (see I and III); however, the difference of charge and mass distributions being small, any exact determination of this value in such a way is difficult for most nuclei. Therefore, to derive the difference between the parameters of total deformation ($\beta - \beta_{\mathrm{e}}$) and between the corresponding radii ($R - R_{\mathrm{e}}$), we shall use the charge oscillations model (III). According to this model the protons may oscillate as a whole inside the nucleus, the energy of these oscillations being found to be connected with the parameters of mass and charge distributions in the nucleus. The low-lying state with $I^\pi K = 1^-0$ in the deformed even–even heavy nuclei is one such charge oscillation. The difference between the parameters of mass and charge distributions can be derived from the energies of these states (III):

$$\beta - \beta_{\mathrm{e}} \simeq 3{\cdot}75\left(\frac{A-Z}{AZ}\right)^{1/2} E_{1^-}{}^{-1/2} A^{-1/3} \tag{8a}$$

$$R - R_{\mathrm{e}} \simeq \frac{1{\cdot}52}{1+0{\cdot}63\beta_{\mathrm{e}}}\left(\frac{A-Z}{AZ}\right)^{1/2} E_{1^-}{}^{-1/2} \tag{8b}$$

where E_{1^-} is the energy of the first level $I^\pi K = 1^-0$ in MeV (we consider R_{e} to be equal to $1{\cdot}216A^{1/3}$ fm†).

The results of the calculations of $\beta - \beta_{\mathrm{e}}$ for the deformed even–even nuclei with $A \geqslant 150$ (when the energies E_{1^-} are known) are given in table 1; the experimental values of E_{1^-} were taken from Begjanov and Rackovyzky (1966) and Backlin *et al.* (1967). As is seen from the results in this paper, the difference in the deformation parameters is not large, although it becomes considerable for some heavy nuclei; this fact is connected with the low positions of the levels with $I^\pi K = 1^-0$ in these cases (see III).

As is seen from equations (5) and (7), the 2^+0-state gyromagnetic factors are independent of the non-axiality parameters and may be derived from the energies E_{1^-} and the parameters β_{e}. Taking into account only the terms of first order of smallness in the expressions for the g_ν factors, we obtain

$$g_{\mathrm{R}}(20) = \frac{Z}{A}\left(\frac{R_{\mathrm{e}}}{R}\right)^2\left(1 - \frac{\beta-\beta_{\mathrm{e}}}{\beta} + 0{\cdot}348\beta_{\mathrm{e}}\frac{\beta-\beta_{\mathrm{e}}}{\beta} + \ldots\right). \tag{9}$$

The results of the calculations of $g_{\mathrm{R}}(20)$ in accordance with equation (9) are given in table 1 for all the deformed nuclei with $A \geqslant 150$ with known energies E_{1^-}. The β_{e} parameter was determined from the experimental data on the quadrupole moments of the nuclei (Dzhelepov 1966, Stelson and Grodzins 1965). The experimental values $g_{\mathrm{R}}(20)$ were taken from the papers by Dzhelepov (1966), Wolfe and Sharenberg (1967), Kurfess and Sharenberg (1967), Münck *et al.* (1966) and Keszthelyi *et al.* (1965). As is seen from table 1, the agreement between the theoretical values of $g_{\mathrm{R}}(20)$ and the available experimental values is satisfactory. Unfortunately, as far as we know the values of $g_{\mathrm{R}}(20)$ have not yet been measured for the actinides, and this fact limits the possibilities for comparison with experiment.

† It is worth noting that in this case one should use a somewhat larger value for R than in IV. However, this specification is not essential for the present as the available data contain considerable experimental errors.

Table 1. The gyromagnetic factors of the first rotational states with $I^{\pi}K = 2^{+}0$ and $2^{+}2$ for the even–even deformed nuclei

Nucleus	E_{1^-} (MeV)	$\beta - \beta_e$	$\frac{Z}{A}$	$g_R(20)$ theor.	$g_R(20)$ exp.	$\frac{\gamma_e}{\gamma}$	$g_R(22)$ theor.	$g_R(22)$ theor. from $g_R(20)$	$g_R(22)$ exp.
^{150}Nd			0·400			3·36		0·67	
^{152}Sm	0·961	0·070	0·408	0·319	0·34 ±0·03	3·42	0·812	0·73	
					0·277 ±0·028				
^{154}Sm	0·927	0·0715	0·403	0·320	0·29 ±0·03	5·33	1·22	1·13	
^{154}Gd	0·9966	0·067	0·416	0·328	0·36 ±0·03	3·55	0·867	0·97	
^{156}Gd	1·366	0·057	0·410	0·338	0·32 ±0·03	3·52	0·888	0·79	
					0·296 ±0·018				
^{158}Gd	0·978	0·0675	0·405	0·326	0·315 ±0·025	4·55	1·08	1·06	
^{160}Dy	1·200	0·0595	0·413	0·336	0·305 ±0·030	3·41	0·860	0·80	
^{162}Dy	1·2754	0·058	0·408	0·336	0·362 ±0·024	4·05	0·999	1·10	
^{164}Dy			0·403			4·12		1·02	
^{164}Er	1·386	0·054	0·415	0·343		3·04	0·794		
^{166}Er	1·663	0·0495	0·410	0·347	0·329 ±0·027	4·14	1·05	1·01	
^{168}Er			0·405			4·28		1·05	
^{170}Er			0·400			3·76		0·93	
^{172}Yb	1·600	0·049	0·407	0·343	0·335 ±0·010	3·86	0·979	0·97	
^{176}Yb			0·398			3·78		0·85	
^{176}Hf	1·722	0·046	0·409	0·344					
^{182}W			0·407			5·35		0·93	
^{186}W			0·398			2·68		0·78	
^{188}Os			0·405			2·0		0·38	0·43 ±0·08
^{194}Pt			0·402			0·65		0·16	0·131 ±0·03
^{222}Ra	0·242	0·104	0·397	0·239					
^{224}Ra	0·217	0·112	0·393	0·223					
^{226}Ra	0·253	0·102	0·390	0·244					
^{226}Th	0·230	0·105	0·398	0·256					
^{228}Th	0·3275	0·088	0·395	0·270					
^{230}Th	0·508	0·071	0·392	0·290					
^{232}Th	1·045	0·0495	0·388	0·314		5·31	1·09		
^{232}U	0·564	0·066	0·397	0·304					
^{234}U	0·790	0·0555	0·394	0·312					
^{236}U	0·688	0·0595	0·390	0·308					
^{238}U	0·679	0·060	0·387	0·305					
^{238}Pu	0·605	0·0625	0·395	0·310					
^{240}Pu	0·597	0·063	0·392	0·308					

The parameters of charge and mass non-axiality are necessary to calculate the 2^+2-state gyromagnetic factors of the non-axial nuclei. For deriving these parameters one may use the calculations, carried out in IV, where the parameters of the mass non-axiality were determined from the energies of the rotational states with $I^\pi K = 2^+0$ and 2^+2, and the parameters of the charge non-axiality were calculated from the ratios of the reduced probabilities of E2 transitions between these levels. The ratios γ_e/γ taken from IV are presented in table 1. The gyromagnetic factor of the 2^+2 non-axial nucleus state, in accordance with equations (7) and (5), is equal to

$$g_R(22) \simeq \frac{2}{3}\frac{Z}{A}\frac{\beta_e}{\beta}\frac{\gamma_e}{\gamma}\left(\frac{R_e}{R}\right)^2\left(1+\frac{1}{2}\frac{\gamma}{\gamma_e}\right) \tag{10a}$$

$$\simeq \frac{2}{3} g_R(20)\left(\frac{\gamma_e}{\gamma}+0{\cdot}5\right). \tag{10b}$$

The results of calculations of $g_R(22)$ according to equation (10*a*) are presented in table 1 in the ninth column. Unfortunately, at present, there are no reliable measurements of $g_R(22)$ for the nuclei with known energies E_1- (for these nuclei the ratios β_e/β and R_e/R can be derived). For this reason the $g_R(22)$ were calculated from equation (10*b*) using the experimental values of $g_R(20)$ (as is seen from table 1, in the cases where the data necessary for both formulae are available, the results obtained from (10*a*) and (10*b*) are close). We know of experiments measuring $g_R(22)$ for only two nuclei (^{188}Os and ^{194}Pt) (Keszthelyi *et al.* 1965, Murroy *et al.* 1967). The agreement of these results with our calculations is quite satisfactory (see table 1).†

It is worth noting that for a number of nuclei, as we have found, the value $g_R(22)$ is equal to or exceeds unity; the experimental verification of this fact seems to be of great interest (in the Davydov–Filippov model the value $g_R(22)$ should be close to Z/A for all nuclei). But one should bear in mind that the values of γ_e/γ were, perhaps, derived with large errors due to the inaccuracy in measuring the ratios $B(E2|00\to22)/B(E2|00\to20)$ (see IV). If more precise values of γ_e/γ are used, the values $g_R(22)$ may change.

3. Probabilities of magnetic dipole transitions between the rotational states of the non-axial nuclei

In this section we shall consider the electromagnetic transitions between the first rotational states with $I^\pi K = 2^+2$ and 2^+0. It is worth noting that in our approach the M1 admixture to E2 transitions between the 2^+2 and 2^+0 rotational states results *from the difference in the nuclear parameters of mass and charge non-axiality.*

Now we shall calculate the values connected with this admixture. The reduced probability of M1 transitions between the 2^+2 and the 2^+0 state is given by

$$B(M1|22\to20) = \frac{9}{20\pi}\sum_{MM'}|\langle 2M'0|\boldsymbol{\mu}_Z|2M2\rangle|^2 \tag{11}$$

M and M' being the Z components of angular momentum in the initial and final states respectively. Using the wave functions from II or IV and taking into account the most significant terms, one obtains

$$B(M1|22\to20) \simeq \frac{1}{2\pi}\left\{\mu_B\frac{Z}{A}\left(\frac{R_e}{R}\right)^2\frac{\beta_e}{\beta}(\gamma_e-\gamma)\right\}^2. \tag{12}$$

† We shall note that, according to Greiner's calculations (Greiner 1966), the value $g_R(22)$ (the 2^+2 state is treated by Greiner (1966) as a γ-vibrational one) should be close to, but always somewhat larger than, $g_R(20)$. This result contradicts the experiment for ^{188}Os and ^{194}Pt.

We shall designate, as usual, the ratio of the intensities of E2 and M1 transition probabilities by δ^2:

$$\left(\frac{\delta}{E}\right)^2 = \frac{1}{E^2}\,\frac{T(E2|22\rightarrow 20)}{T(M1|22\rightarrow 20)}$$

where E is the transition energy. Using equation (12) and the results for $B(\mathrm{E}2|22\rightarrow 20)$ from IV and considering only the most significant terms, we obtain

$$\frac{\delta}{E} \simeq 0{\cdot}352\frac{A}{Z}Q_0\left(\frac{R}{R_e}\right)^2\frac{\beta}{\beta_e}\left(1-\frac{\gamma}{\gamma_e}\right)^{-1} \tag{13}$$

where Q_0 is the intrinsic quadrupole moment. The results of the calculations of $\lg(\delta/E)^2$ for the nuclei with known energy E_{1^-} are represented in figure 1 by the full line.

To increase the possibility of comparison of $\lg(\delta/E)^2$ with experiment the values β/β_e were also determined from the experimental values $g_R(20)$ using equation (9). In this case δ/E may be written approximately as follows:

$$\frac{\delta}{E} \simeq 0{\cdot}352 Q_0\frac{1}{g_R(20)}\left(1-\frac{\gamma}{\gamma_e}\right)^{-1}. \tag{14}$$

The results of the calculations with equation (14) are represented in figure 1 by the broken line. The experimental data are represented in figure 1 by triangles and arrows (the arrows mark the highest and the lowest limits of the quantity) and are taken from the paper by Greiner (1966). The nucleus ^{232}Th is the only one among the actinides for which the experimental data are available to calculate $\lg(\delta/E)^2$. In this case our calculations give $\lg(\delta/E)^2 = 2{\cdot}23$.

The results of Greiner's calculations (Greiner 1966, one of the two calculated curves) are represented by the broken line with crosses.

As is seen from figure 1, our results prove to be rather close to Greiner's results, though the models used are different.

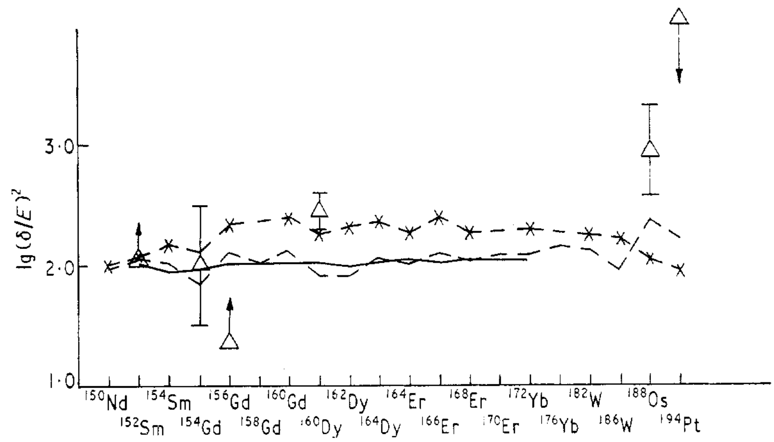

Figure 1. Relative intensities of E2 and M1 transitions between the 2^+2 and 2^+0 rotational states. The full line represents the results of calculations in accordance with equation (13), the broken line those in accordance with equation (14). The experimental data are represented by triangles and arrows. The broken line with crosses represents the results of Greiner (1966).

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